

**Problems: Total from area density**

1. Charge is distributed on a flat rectangular region of dimensions  $L$  by  $W$  so that the area charge density is proportional to the distance from one corner, reaching a maximum of  $\sigma_0$  at the far corner. Set up an iterated integral to compute the total charge  $Q$ . You do not need to evaluate this integral. As an optional challenge, you can try to evaluate the iterated integral.

$$\text{Answer: } Q = \frac{\sigma_0}{\sqrt{L^2 + W^2}} \int_0^W \int_0^L \sqrt{x^2 + y^2} \, dx dy$$

2. Consider a thin rectangular plate of dimensions  $L$  by  $W$  with uniform thickness. The materials composing the plate vary from point to point so that the area mass density is proportional to the square of the distance from the center of the plate, reaching a maximum of  $\sigma_0$  at each of the four corners. Compute the total mass  $M$  of the plate.

$$\text{Answer: } M = \frac{1}{3}WL\sigma_0$$

3. Charge is distributed on an isosceles triangle of height  $H$  and base length  $B$  so that the area charge density is proportional to the distance from the base, reaching a maximum of  $\sigma_0$  at the vertex opposite the base. Compute the total charge  $Q$ .

$$\text{Answer: } Q = \frac{1}{6}BH\sigma_0$$

4. Charge is distributed on an equilateral triangle of side length  $L$  so that the area charge density is proportional to the square of the distance from the center of the triangle, reaching a maximum of  $\sigma_0$  at each of the three vertices. Compute the total charge  $Q$ .

5. Charge is distributed on the surface of an open right circular cylinder of radius  $R$  and height  $H$  so that the area charge density is proportional to the distance from one end of the cylinder, reaching a maximum of  $\sigma_0$  at the other end. Compute the total charge  $Q$ .

Note: This situation involves an area density. Depending on how you approach the problem, you might end up constructing and evaluating an integral in just one variable.

$$\text{Answer: } Q = \pi RH\sigma_0$$